

The New Weibull Handbook

**Reliability & Statistical Analysis for Predicting Life, Safety,
Risk, Support Costs, Failures, and Forecasting Warranty
Claims, Substantiation and Accelerated Testing,
Using Weibull, Log Normal, Crow-AMSAA ,
Probit, and Kaplan-Meier Models**

**Fifth Edition by Dr. Robert B. Abernethy
Author and Publisher**

Dedication:

This book is dedicated to my wife, Sally, for her encouragement and patience. Further, it is dedicated to all my friends and colleagues that helped so much. Finally, it is dedicated to the reader with the hope it will have "some utility," to use Professor Weibull's words.

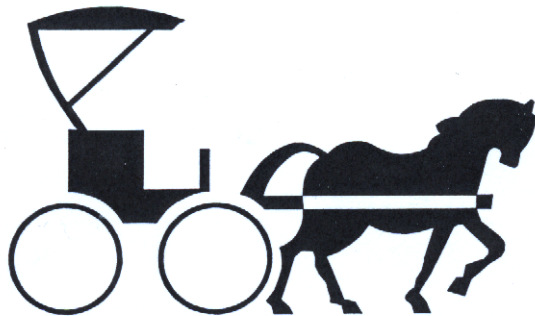
And lastly we have a special note for the Weibull Workshop students for whom we designed this Handbook...

Weibull Workshop Students:

If you are fortunate enough to have obtained or borrowed the Handbook before your Workshop, studying the Handbook before the Workshop is very helpful. **Please** read at least Chapters 1-4. We will cover the basic standard method the first day, Chapters 1-4. For the second day, study Chapters 5-7 (7 is least important). For the third day, Chapters 8, 9, & 10. (Most important material for the third day is the Crow-AMSAA Model.) Chapter 11 is case studies which you should browse. Please bring a clear plastic ruler, a calculator, and a problem from your work. The data should be in a single column, positive values are failures, negative are suspensions. You may bring the data in ASCII format, or a spreadsheet file (Excel) or WinSMITH Weibull files. The file may be on a diskette or in your laptop.

"Now in building of chaises, I tell you what,
There is always *somewhere* a weakest spot,-
In hub, tire, felloe, in spring or thill,
In panel, or crossbar, or floor, or sill,
In screw, bolt, throughbrace,-lurking still,
Find it somewhere you must and will,-
Above or below, or within or without,-
And that's the reason, beyond a doubt,
A chaise *breaks down*, but doesn't *wear out*.

Oliver Wendell Holmes... "The One-Hoss Shay"



PREFACE

Background A quarter of a century has passed since the *USAF Weibull Analysis Handbook* was published. In the years that followed, the author frequently taught Weibull analysis using the USAF Handbook as the text, encouraged by both the United States Air Force and Pratt & Whitney. In 1987, the author retired early to devote full time to research and lecturing in Weibull Analysis. It has been most enjoyable and interesting. The best part has been meeting outstanding engineers from many industries with new and different applications for Weibull methods. The students inspire additional research and methods development. This in turn required rewriting and updating the USAF Handbook producing *The New Weibull Handbook*. This document is the fifth edition. It contains the latest methods, techniques, and applications that have been developed by the author, Wes Fulton, Paul Barringer, Joe Wauben, Bob Rock, Wayne Nelson, Randy Pirtle, Todd Marquart, Carl Tarum, Geoff Cole and David Navell and many others.

The New Weibull Handbook has become the national and international standard for Weibull analysis. It is in daily use throughout the world. Thousands of copies have been distributed. Users include military, automotive, medical, electronics, materials & composites research, aerospace, electrical power, nuclear power, dental research, advertising, bearings, compressors, v-belts, chain drives and on and on. Many organizations have adopted the Handbook and software implementing its methods as standard including the Federal Aviation Administration, Harley Davidson, Motorola, TVA, Honeywell, BICC General, GKN Automotive, Dupont, Meritor Automotive, Teleflex, Guidant Cardiac Pacemaker (CPI), Daimler Chrysler, General Electric, Siemens, Behr, Borg-Warner, NASA, Rolls Royce, Gates Rubber, the US Naval Air Arm (NAVAIR), the US Air Force at SAALC and WPAFB, and Dow Chemical. The author has provided many free copies (shipping charges only) of the Handbook to universities in Europe, North America, and Australia. The author hereby extends this offer to university libraries worldwide. Wes Fulton provides free demonstration copies of the SuperSMITH software on his web site.

With this fifth edition there is a new companion text, "The New Weibull Handbook Student Version." It includes an introductory version of the WinSMITH Weibull Software. The author provides free copies (shipping charges only) of the student handbook and the student software to professors and lecturers that teach reliability as well as to university libraries that request copies.

The **SuperSMITH** software is the most significant technical breakthrough. Wes Fulton has created friendly, easy-to-use software that does everything in the Handbook. *The New Weibull Handbook* provides an understanding and reference for SuperSMITH. The computer software has made application so much easier. It has also made teaching Weibull easier, more efficient, and more fun. Within the Weibull workshops, many more applications and case studies can be treated without the drudgery of hand calculations and plotting. As this document is the workbook for the Weibull Workshops, many references to the software capabilities are included. This referencing may appear to be inappropriate, but the reliance on software in the workshops requires it. If the student using *The New Weibull Handbook* desires, he may avoid almost all hand calculations and hand plotting by using SuperSMITH. The new "*Playtime with SuperSMITH*" computer tutorial enhances the Handbook and provides many step-by-step case studies illustrating all the methods. Alternatively, the reader may use this Handbook to write "home-grown" software. Outputs are easily validated with benchmark case studies found in both demonstration and full versions of the SuperSMITH software.

The **Fifth Edition** features new technology:

- Goodness of fit, illustrating for the first time the "power" of the three best methods for goodness of fit: Critical Correlation Coefficient, likelihood ratio and Anderson-Darling. "Power" is the probability that the method makes the correct choice identifying the parent distribution for a give data set.
 - Crow-AMSAA modeling has become so popular that it has been moved to a new chapter. The new write-up includes the latest methods from the new IEC standards with best practices recommended.
 - A new sections on warranty analysis has been added by Joe Wauben of Honda
 - Life Cycle Costs are presented in a new section by Paul Barringer.
 - Wayne Nelson has updated his on graphical repair analysis with a case study in Appendix M.
 - Bob Rock of PACCAR updated his extremely useful life data analysis logic diagram which takes you step-by-step to the best methods and analysis for your particular data. The logic diagram is also available in Spanish.
-

- Todd Marquart of Micron Technology has completed an extensive comparison of the three methods for interval and grouped data. Which is most accurate?
- The New Weibull Handbook is unique in recommending “best practices” wherever there are alternative methods available. Some of the “best practices” have changed based on new research.
- Cost analysis is described for production process reliability, failure forecasts, Crow-AMSAA, Life Cycle Costs and optimal parts replacement.

To make the Handbook more readable, all the heavy mathematics will be found in the appendixes. Asterisks on section subtitles indicate advanced topics. The novice reader may skip these advanced sections. As the workbook is designed for the Weibull Analysis Workshops, the format includes much white space for notes and calculations. Dozens of case studies are employed to present the methodology. Paul Barringer has taken the most important reference texts and papers from my library and put them on his Website for downloading. <http://www.barringer1.com> They are indicated by a \$ sign preceding references throughout the Handbook..

Thank You: Many friends, students, experts, and clients have helped and contributed much to the Handbook. For me, all this support has been overwhelming. Everyone helped! Thank you all. Some are mentioned here:

- First, my associate, Wes Fulton, not only for his friendly software, but also for his research and development of new Weibull methods.
- Second, our associate, Paul Barringer, for his help, comments, reviews, research, and contributions.
- A board of industry experts has reviewed the fifth edition and provided many comments and contributions: Walter Thomas, Wes Fulton, Ronald Schop, Bob Rock, Charles DeSica, Randy Pirtle, Todd Marquart, Carl Tarum, Geoff Cole, Paul Barringer, Jim Breneman, and Joe Wauben. I am indebted to them.
- Many contributed sections of the Handbook: Bob Rock, PACCAR; David B. Smith, Detroit Edison; Bob Scanlon, Southern Pacific (now with the US Postal Service); Sten-Åke Irell, Volvo Flygmotor; Marie Stanton, FPL; David Weber, Consultant; Carl Tarum, Delphia; Hans Iwand, Union Pacific; Drs. Memis and Lautenschlager, Dentists at Northwestern University; Richard Rudy, Chrysler; Kenneth Young, Hydri; Jim Keener and Mel Thomas, FPL; Bill Pritchard, TVA; Shashank Kolhatkar, Borg Warner; Charlie Williams, Florida Power; Geoffrey Cole and David Navell, Rolls Royce; David Williamson, Dow Chemical; M.A. Vasan, Meritor Automotive; David B. Smith, Detroit Edison; and David Langanke and Randy Pirtle, Honeywell Engines.
- This edition is improved by incorporating the suggestions from the excellent constructive review of the Handbook by Dr. Alan Winterbottom for the Royal Statistical Society Journal A (1997), 160, Part 2. The author is indebted.
- None of the above are in any way responsible for errors in the Handbook. The author takes all the blame.

Please call or write with questions, comments, and constructive criticism about *The New Weibull Handbook*. I would like to hear from you: E-mail is preferred: weibull@att.net

Dr. Robert B. Abernethy
Wintertime: 536 Oyster Road
North Palm Beach, Florida 33408-4328
Phone: 561-842-4082

Summertime: 174 Tanasi Lagoon Drive
Loudon, Tennessee, 37774-2984
Phone: 865-408-0541

Table of Contents

Chapter 1	An Overview Of Weibull Analysis	1-1
1.1	Objective	1-1
1.2	Background	1-1
1.3	Examples	1-2
1.4	Scope	1-2
1.5	Advantages of Weibull Analysis	1-3
1.6	Data, Discrete Versus Life Data	1-3
1.7	Failure Distribution	1-4
1.8	Failure Forecasts And Predictions	1-5
1.9	Engineering Change Test Substantiation	1-6
1.10	Maintenance Planning	1-6
1.11	System Analysis And Math Models	1-7
1.12	Weibulls With Curved Data	1-7
1.13	Weibulls With Corners And Doglegs	1-9
1.14	Weibayes	1-9
1.15	Small Sample Weibulls	1-9
1.16	Updating Weibulls	1-9
1.17	Deficient (Dirty) Data	1-9
1.18	Establishing The Weibull Line, Choosing The Fit Method	1-10
1.19	Related Methods And Problems	1-10
1.20	Summary	1-11
Chapter 2	Plotting The Data And Interpreting The Plot	2-1
2.1	Foreword	2-1
2.2	Weibull Data	2-1
2.3	The Weibull Plot Scales	2-2
2.4	η (Eta) and β (Beta)	2-2
2.5	Weibull Analysis - An Example	2-4
2.6	Median Ranks	2-5
2.7	The Weibull Plot	2-6
2.8	"B" Life	2-6
2.9	Suspended Test Items	2-7
2.10	Bernard's Approximation	2-7
2.11	Suspensions Increase Eta	2-8
2.12	Interpreting The Weibull Plot	2-8
2.13	Beta < 1 Implies Infant Mortality	2-9
2.14	Beta = 1.0 Implies Random Failures	2-10
2.15	1.0 < Beta < 4.0 Implies Early Wear Out	2-11
2.16	Beta > 4.0 Implies Old Age (Rapid) Wear Out	2-11
2.17	Weibull Modes May Be "Covered"	2-12
2.18	Weibull Paper And Its Construction	2-12
2.19	Weibull Analysis - The Standard Method	2-14
2.20	Problems	2-13
Chapter 3	Dirty Data, "Bad" Weibulls, And Uncertainties	3-1
3.1	Foreword	3-1
3.2	Small Sample Uncertainties	3-1

3.2.1 Goodness Of Fit	3-3
3.3 Suspensions	3-6
3.4 Suspect Outliers	3-6
3.5 Curved Weibulls And The t_0 Correction	3-7
3.6 Curved Weibulls And The Log Normal Distribution	3-11
3.7 Data Inconsistencies And Multimode Failures	3-14
3.7.1 Low-Time Failures	3-14
3.7.2 Close Serial Numbers	3-15
3.7.3 Mixtures Of Failure Modes	3-16
3.8 Steep Slopes Hide Problems	3-17
3.9 Bad Weibull Patterns	3-18
Conclusion	3-18
3.10 Problems	3-19
Chapter 4 Failure Forecasting = Risk Analysis	4-1
4.1 Situation	4-1
4.2 Definition	4-1
4.3 Forecasting Techniques	4-1
4.4 Calculating Failure Forecasts	4-1
4.4.1 Expected Failures Now	4-1
4.4.2 Failure Forecast When Failed Units Are Not Replaced	4-3
4.4.3 Failure Forecasts When Failed Units Are Replaced	4-3
4.5 Failure Forecast Analysis-Summary	4-4
4.5.1 Case Study 1: Bearing Cage Fracture	4-5
4.5.2 Case Study 2: Bleed System Failures	4-7
4.6 System Failure Forecast Without Simulation*	4-12
4.6.1 Case Study 3: Aircraft In-Flight Engine Shutdowns*	4-12
4.7 System Failure Forecasts With Simulation* 4-15	
4.7.1 Case Study 4: System Failure Forecast With Simulation*	4-17
4.8 Optimal (Lowest Cost) And Block Replacement Intervals*	4-19
4.9 Problems	4-25
Chapter 5 Maximum Likelihood Estimates & Other Alternatives	5-1
5.1 Introduction	5-1
5.2 Maximum Likelihood Estimation (MLE)	5-1
5.3 MLE With Reduced Bias Adjustment (RBA) for Accurate Results	5-3
5.3.1 The RBA Factor for Normal and Lognormal Distributions	5-4
5.3.2 The RBA factor for the Weibull distribution	5-5
5.3.3. Best Practice	5-6
5.4 Median Rank Regression: X on Y Versus Y on X	5-7
5.5 Plotting Positions	5-9
5.5 Special Methods: Mle With Reduced Bias Adjustment (Rba)	5-6
5.6 Special Methods: Gossett's Student's T	5-10
5.7 The Dauser Shift - Unknown Suspension Times	5-10
5.8 Special Methods For Inspection Interval Data And Coarse Data	5-12
5.8.1 Inspection Option #1	5-12
5.8.2 & 5.8.3 Probit Analysis Inspection Options #2 & 3	5-13
5.8.4 Kaplan-Meier (KM) Inspection Option #4	5-14
5.8.5 Interval Maximum Likelihood Estimation (MLE) Inspection Option #5	5-14
5.9 Distribution Analysis	5-16

Chapter 6	Weibayes And Weibayes Substantiation Testing	6-1
6.1	Foreword	6-1
6.2	Weibayes Method	6-2
6.3	Weibayes Without Failures	6-2
6.4	Weibayes With Failures	6-3
6.5	Unknown Failure Times	6-4
6.6	Weibayes Worries And Concerns	6-4
6.7	Weibayes Case Studies	6-5
6.8	Substantiation And Reliability Testing	6-9
6.9	Zero-Failure Test Plans For Substantiation Testing	6-10
6.10	Zero-Failure Test Plans For Reliability Testing	6-12
6.10.1	Re-Expression Of Reliability Goal To Determine ?	6-12
6.10.2	Tailoring and Designing Test Plans	6-14
6.11	Total Test Time	6-15
6.12	Test-To-Failure Versus Weibayes Zero-Failure Tests	6-16
6.13	One Or Zero Failure Test Plans	6-19
6.14	Sudden Death Tests With Weibull And Weibayes	6-20
6.15	Case Study: Cost Vs. Uncertainty Trades	6-23
6.16	Normal And Lognormal Tests	6-22
6.17	Accelerated Testing	6-24
6.17.1*	Accelerated Step-Stress Test Data Analysis	6-25
6.17.2*	Accelerated Testing: A Method For Estimating Test Acceleration Factor With No Existing In-Service Failures	
6.18	System Deterioration	6-28
6.19	Weibull Libraries And Lessons Learned	6-29
6.19.1	A Weibull Library Case Study	6-30
6.19.2	Weibull Libraries For End Users	6-31
6.21	Problems	6-32
Chapter 7	Interval Estimates	7-1
7.1	Interval Estimates	7-1
7.2	Confidence Interval Concept	7-1
7.3	Confidence Intervals For B Lives And Reliability	7-2
7.3.1	Beta-Binomial Bounds	7-3
7.3.2	Fisher's Matrix Bounds	7-4
7.3.3	Likelihood Ratio Bounds	7-5
7.3.4	Pivotal Bounds Monte Carlo Bounds	7-6
7.3.5	Reliability Assurance Interval and the "p" value	7-7
7.3.6	Normal Distribution Confidence Bounds with Student's t	7-7
7.3.7	Summary Of Confidence Bounds For B Life And Reliability	7-8
7.4	Confidence Intervals For Eta And Beta	7-8
7.5	Are Two Weibull Data Sets Different Or From The Same Distribution	7-9
7.5.1	Double Confidence Bounds Do Not Overlap	7-10
7.5.2	Likelihood Ratio Test	7-11
7.5.3	Likelihood Contour Plots	7-11
7.6	Problems - True Or False?	7-13
Chapter 8	Related Math Models	8-1
8.1	Introduction	8-1
8.2	Binomial Distribution	8-1
8.3	Poisson Distribution	8-5
8.4	Binomial Becomes Poisson... Sometimes	8-9
8.5	The Exponential Distribution	8-11

8.6 Kaplan-Meier Survival Estimates	8-12
8.7 Probabilistic Design	8-17
8.7.1 Strength-Load And Life-Usage Interactions	8-17
8.7.2 Total Life = Crack Life + Crack-To-Rupture Life	8-18
8.7.3 Does Failure Mode A Cover Mode B?	8-19
8.8 Production Process Reliability	8-19
8.9 Extreme Value Statistics	8-21
8.10 Batch Effects	8-23
8.11 Problems	8-24

Chapter 9 Crow-AMSAA Modeling, Warranty Analysis & Life Cycle Costs 9-1

9.0 The Crow-AMSAA-Duane Reliability Growth Model	9-1
9.1 Background History	9-2
9.2 CA Methods	9-2
9.2.1 Simple Graphical and Regression Solution	9-2
9.2.2 IEC Solutions for Time and Failure Terminated Data	9-5
9.2.3 IEC MLE Solutions for Interval and Grouped Data	9-7
9.3 Comparisons of the IEC and Regression CA Methods	9-12
9.4 CA Input May Be Confusing	9-14
9.5 Missing Early Data with CA	9-14
9.6 First Time Failure Mode Analysis	9-14
9.7 Warranty Claims Analysis	9-15
9.8 Warranty Data Matrix	9-16
9.9 Warranty Data Rules	9-17
9.10 Warranty Data Matrix Conversion and Analysis	9-18
9.11 Warranty Analysis Methods	9-20
9.11.1 Inspection Option #1	9-20
9.11.2 Kaplan-Meier	9-20
9.11.3 MLE Interval	9-20
9.11.4 Crow AMSAA	9-20
9.12 Case Studies	9-21
9.13 Tracking Your Results	9-21
9.14 Warranty Conclusions and Recommendations	9-21
9.15 Life Cycle Cost	9-21
9.16 Net Present Value (NPV)	9-21
9.17 Discount Rate	9-22
9.18 Life Cycle Cost and NPV	9-22
9.19 LCC Calculations	9-23
9.20 Case Studies	9-24

Chapter 10 Summary 10-1

10.1 The Beginning Of The End	10-1
10.2 Which Method? What Kind Of Data?	10-1
10.3 Looking At The Plot, What Do You See?	10-3
10.4 Which Distribution Is Best?	10-4
10.5 Substantiation And Accelerated Testing	10-6
10.6 Confidence Intervals	10-6
10.7 Presentations And Reports	10-6
10.8 Logic Diagram - Flowchart	10-6
10.9 The End	10-6
10.10 Best Practice Flow Chart	10-7

Chapter 11 - Case Studies And New Applications

11.1 Foreword	
11.2 Stress Corrosion Failure Forecasting	11-2
11.3 Optimal Component Replacement - Voltage Regulators	11-3
11.4 Locomotive Power Units Overhaul Life	11-7
11.5 Low Cycle Fatigue Cracks In Turbine Disks	11-8
11.6 Cost Effective Calibration Intervals	11-10
11.7 Machine Tool And Robotic Accuracy	11-12
11.8 Shear Ram Blowout Preventer Tests	11-13
11.9 Florida Power & Light Turbogenerator Failure	11-17
11.10 TVA Bull Run Fossil Plant - Controller Cards	11-18
11.11 Are The Bonded Drives Failing Because Of Cold Weather, Or Just Plain Worn Out?	11-19
11.11.1 Introduction	11-19
11.11.2 Background And Analysis	11-20
11.11.3 Results	11-21
11.11.4 Conclusions And Recommendations	11-23
11.12 Repairable Systems Reliability Growth Assessment	11-24
11.13 Front Jounce Bumpers	11-25
11.14 Transfer Case Seal	11-26
11.15 Dental Acrylic Adhesive Fatigue	11-27
11.16 Duane-Crow-Amsaa Reliability Modeling	11-28
11.17 Weibull Analysis Of Boiler Tube Failures	11-31
11.18 Gas Turbine Seal Failures - A Batch Problem	11-34
11.19 Covering Warranty Claims In Weeks To Miles	11-36
11.20 Challenger Space Shuttle Weibull	11-37
11.21 CA Test-Fix-Test Reliability Growth for Truck Engines	11-38
11.22 Crow-AMSAA Analysis New Failure Modes	11-41

Appendix A: Glossary A-1

Appendix B: Rank Regression And Correlation Method Of Weibull Analysis B-1

B.1 Method	B-1
B.2 Example And Step-By-Step Procedure	B-1

Appendix C: Maximum Likelihood Estimation* C-1

C.1 Foreword	C-1
C.2 Statistics, Probability And Likelihood	C-1
C.3 The Likelihood Function	C-1
C.4 Maximizing The Likelihood Function	C-2
C.5 Maximum Likelihood Example	C-3
C.6 Interval MLE	C-6
C.7 Maximum Likelihood Versus Median Rank Regression Estimates	C-8

Appendix D: Goodness of FitD-1

Appendix E: Weibayes Analysis E-1

E.1 Foreword	E-1
E.2 Weibayes Equation With No Failures	E-1
E.3 Weibayes With Failures	E-2

Appendix F: Batch Failures Using The Aggregated Cumulated Hazard Function F-1

- F.1 Batch Failures On Weibull Plots F-1
- F.2 Batch Problems With The "Present-Risk" Method F-2
- F.3 The Ach Method F-3
- F.4 A Case Study: Aero-Engines – (Lp Turbine Strap Failures) F-4
- F.5 Concluding Remarks F-5

Appendix G: Weibull And Log Normal Mean And Variance G-1

- G.1 Rth Moments G-1
- G.2 Weibull Mean G-2
- G.3 Weibull Variance G-3
- G.4 Weibull Mode G-3
- G.5 Weibull Median G-3
- G.6 Log Normal Mean And Standard Deviation G-3
- G.7 Log Normal Variance G-4

Appendix H: Weibull Graph Paper H-1

Appendix I: Median Ranks I-1

Appendix J – Mixtures Of Populations And Failure Modes J-1

- J.1 Competing Risk: J-1
- J.3 Competing Risk Mixture: J-2
- J. 4 Compound Competing Risk Mixture: J-2
- J.5 Weibath Model: J-2
- J.7 Curve Shape. J-3

Appendix K: Answers To Problems K-1

Appendix L: The C4 Factor L-1

Appendix M: Graphical Repair Analysis M-1

Appendix N: Waloddi WeibullN-1

References R-1

Index I-1
